Frequency dependence of Bloch impedance in a periodic transmission line structure

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Abstract - Frequency dependence of Bloch impedance in a periodic structure was calculated by using simple F-matrix formula. Also, relationship between band-gap on-set frequency and reactance factor was evaluated. Finally, formula was checked by the experiment.

Ⅰ. INRODUCTION

Electronic band gap structure is well investigated in solid-state physics [1]. For a couple of decades ago, also the existence of photonic band gap (PBG) phenomenon was theoretically proved for an artificial lattice [2]. Looking into the microwave field, filter theory and theory of propagation phenomenon in periodic wave-guide structure have been already well established [3],[4]. In recent years, although, the propagation phenomenon under periodically modulated structure has investigated rigorously again from the aspect of photonic band gap concept. In fact, many of novice ideas have been proposed in such approaches.

For example, photonic band gap structures were applied to power amplifier matching circuit for enhancing the power added efficiency [5], planar antenna for enhancing its efficiency [6] and novice phase shifters [7], [8]. For power amplifier matching circuit application, high reflection property caused by PBG band stop region was used for harmonic termination to enhance power added efficiency [5]. Though phase response was not treated explicitly in such papers, it is important for acquiring high power added efficiency. For example, for the F-class amplifier output matching circuit, short response is required for even harmonics and open response is required for odd harmonics.

For phase shifter application, dispersion-shift phenomena caused by the periodically loaded structure are utilized, where MEMS capacitors [7] or varactor diodes [8] were adopted for periodically loading. In such systems, periodically loaded reactance values are a function of a driving voltage. Then, dispersion relation can be dynamically controlled. This interesting feature can be used not only for the phase shifter but also the tunable filters or another adaptive devices used in intelligent systems such as Software Defined Radio, which is assumed as one of the key technology for next generation of wireless communication. The phase shift phenomena have been treated well, but characteristic impedances of nonlinear transmission line have not been treated so far.

Also, above-mentioned researches were made basically by electro-magnetic simulation or experimental approach. For manufacturing point of view, we need an analytical form for anticipating the electrical parameters for the designing.

In this paper, the dispersion relation is reviewed by using simple F-matrix formula. Then, band gap on-set frequency is calculated by utilizing the formula. Next, the frequency dependence of Bloch impedance is shown, which has very interesting feature and has not been discussed in detail so far.

Finally, experimental result validated the effectiveness of the model.

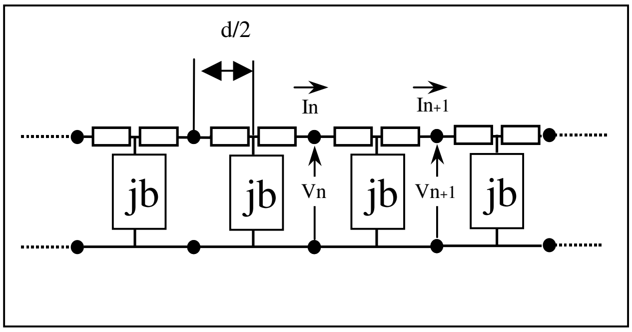

Fig. 1 Equivalent circuit of periodically loaded transmission line

Ⅱ. DISPERSION RELATION

Figure 1 shows a kind of stereotypical equivalent circuit of periodically loaded transmission line. Reactance factor jbs are loaded periodically in a shunt topology. Length of the transmission line in unit cell is d. Characteristic impedance of the unloaded line is Zo.

With using these parameters, dispersion relation of the periodically loaded line is calculated straightforward. The F-matrix of the unit cell in Fig. 1 is as follows;

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos(kd) - \frac{bZo}{2}\sin(kd) & j(Zo\sin(kd) + \frac{bZo}{2}\cos(kd) - \frac{bZo^2}{2}) \\ j(\frac{1}{Zo}\sin(kd) + \frac{b}{2}\cos(kd) + \frac{b}{2} & \cos(kd) - \frac{bZo}{2}\sin(kd) \end{bmatrix} \quad (1)$$

where k is free space wave number, d is unit line length, b is reactance value and Zo is line characteristic impedance.

Whereas, periodic boundary condition is expressed as,

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$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} V_{n+1} e^{\beta d} \\ I_{n+1} e^{\beta d} \end{bmatrix}, \qquad (2)$$

where β is propagation constant of the mode. We treat the loss less case, then we set propagation constant $\gamma = \beta$ in equation (2). Substitute eq. (1) into eq. (2), then we get dispersion relation of periodically loaded transmission line;

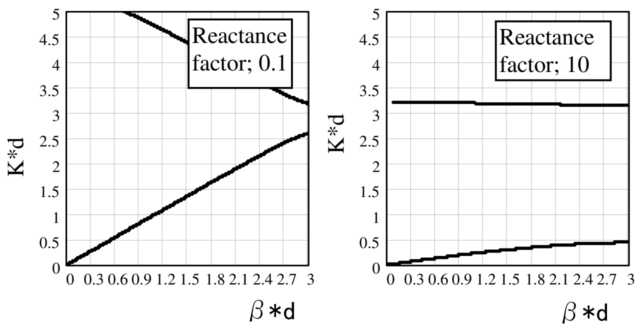
$$\cos(\beta d) = \frac{A+D}{2}\cos(kd) - \chi(kd)\sin(kd) \quad , \qquad (3)$$

where χ is the reactance factor, which is described as follows;

$$\chi = \frac{CoZoc}{2d} . \qquad (4)$$

In equation (4), Co is a capacitance value; c is the speed of light. In equations (3) and (4), reactance value b was replaced by ωCo [9].

Figure 2 shows the dispersion relation of first and second band in the case of reactance factor of 0.1 and 10 by the equation (3).



(a) (b)

Fig.2 K- β diagram for a periodically loaded transmission line. Reactance factor is 0.1 for (a) and 10 for (b) in equation (3).

Compare the case in reactance factor of 10 (Fig.2 (b)) to the case in that of 0.1 (Fig.2 (a)), band gap at the Brillouin zone was 5 times enlarged. For industrial point of view, the device size should be as small as possible. And adopting high reactance value for the periodic loading is the one of the effective technique to miniaturize the device. First Brillouin zone locates at half wavelength. And we have to have at least 3 or more periods - of course, it depends on how much you need the sharpness of filtering response - to be able to regards the structure as periodic. This restriction does not affect for optical application seriously, where wavelength is small enough for the device size we want. But for the microwave and millimeter-wave application, this restriction degrades the device usefulness in some situation for its size. When low pass filtering feature is required, that is to say, we want to realize the stop band at the lowest region without enlarging the device, it is very effective to use larger reactance value as periodical loads [10]. In such point of view, the interesting point is to find the dependency of on-set band gap frequency change with increasing reactance (capacitance) value.

To clarify the relationship between band gap on-set frequency and reactance factor, which was defined in eq. (4), we substitute βd=π in eq. (3).

$$-1 = \cos y - \chi y \sin y \quad , \qquad (5)$$

where kd was replaced by y for the simplicity.
By simple transformation, we get,

$$\chi = \frac{1}{y \tan \frac{y}{2}} \qquad (6)$$

In fig. 3, solid line shows the relationship between the reactance factor and k*d for the first band, and dotted line shows that of the second band. When reactance factor is zero, which is the case of unloaded line, k*d becomes π for both bands, that means there is no band gap. With increasing the reactance factor, k*d decreases, which means that band gap starts at lower frequency. The second band goes down quickly with increasing reactance factor compare to the first band, which means we can get sharper pass band filter with higher bands.

For example, we treat the case that d equals to 1 mm and characteristic impedance equals to 50 ohms. From equation (4),

$$Co = \frac{\chi\sqrt{\varepsilon_r}\cdot 2\cdot 10^{-3}}{50\cdot 2.9979\cdot 10^{8}} = 0.133\cdot\chi\cdot\sqrt{\varepsilon_r}\,(pF) \qquad (7)$$

If we want to shift the first band stop point from 3.14 (non band gap limit) to 1, from fig.3 we understand that we have to set reactance factor χ = 2. When alumina or GaAs substrate is used, relative effective dielectric constant εr is around 6.7 with using coplanar wave-guide and realize 50 ohm line , in this case, χ = 2 corresponds to the capacitance of 7 pF.

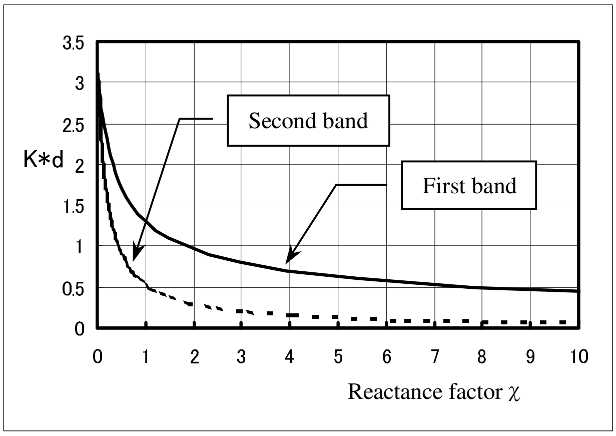


Fig. 3 Relationship between band gap on-set point in first / second bands and reactance factor.

Ⅲ. BLOCH INPEDANCE

Characteristic impedance in periodically loaded line is called Bloch impedance. It has nearly the same meaning as image impedance [3]. A periodic structure has interesting frequency response in such as a propagation constant, a transmission response and so on and investigated deeply. Though, the frequency response of characteristic impedance is not investigated in explicit way so far. In this section, we evaluate the frequency response of characteristic (Bloch) impedance by the model discussed above.
From equation (2), we get,

$$\left(A - e^{\beta d}\right)\cdot V_{n+1} + B \cdot I_{n+1} = 0 \tag{8}$$

By using equation (1) and (8), we get,

$$Z_B^{\pm} = \frac{\pm B}{\sqrt{A^2 - 1}} \quad , \tag{9}$$

where A and B are components of F-matrix which are expressed in equation (1). In deriving equation (9), we assume that unit cell is symmetrical, that is to say A = D [9].

Figure 4 shows the Frequency response of the Bloch impedance. For the calculation, we took reactance factor χ as 0.1. Then we can compare fig. 4 to fig.2 (a), though in fig.2 (a), only first two bands were indicated where up to the middle of 4th pass band were shown in fig. 4. At low frequency limit, Bloch impedance has taken the same value as ordinary characteristic impedance, which value is the geometrical average of unloaded line part and loaded line part. In the first band, Bloch impedance becomes lower with increasing frequency and decreases rapidly near the band edge. As we treat loss less transmission line, Bloch impedance is pure real in the pass band. Bloch impedance becomes zero at the edge of the band gap. Beyond the edge, Bloch impedance becomes pure imaginary, which means the mode becomes evanescent. With increasing frequency, Bloch impedance becomes pure real again, which means the pass band again. Through fig. 4, some interesting phenomena are seen.

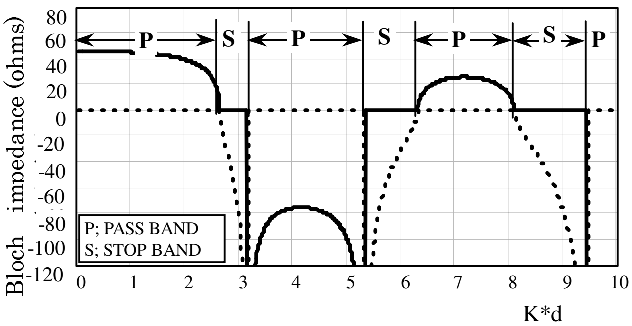


Fig. 4 Response of Bloch impedance over multi bands
Solid line shows real part of Bloch impedance.
Dashed line shows imaginary part of that one.

1) IN ODD PASS BAND;
At the edges of the bands, Bloch impedances become zero.
At the middle of the bands, Bloch impedances become smaller with increasing an ordinal number of band.

2) IN EVEN PASS BAND;
At the edges of the bands, Bloch impedances approach infinity. At the middle of the bands, Bloch impedances become larger with increasing an ordinal number of band.

The +/− solutions in equation (9) correspond to the characteristic impedance for forward and backward traveling wave respectively.

Ⅳ. MEASUREMENT RESULT

To validate the model effectiveness, measurement was made. For taking into account of easiness of future mass production, coplanar wave-guide structure was adopted. Parallel plate-type chip capacitors were mounted on alumina substrate periodically. Ribbon wires connect capacitors to the ground planes on both ends electrically. Ribbon wire and coplanar structure minimize the shunt inductance, so that we neglect it in the relevant frequency. The sample dimensions were listed in table 1.

Table 1 Sample dimensions and physical parameters

***Transmission line**
Line type; coplanar wave guide
Line width; 400 microns
Gap ; 160 microns
*** Substrate**
Height ; 635 microns
Relative permittivity; 9.8(alumina)
*** Periodical loads**
Period length ; 5mm
Capacitance ; 1 pF
Num of periods ; 5

In fig. 5, solid line shows the simulated data, whereas dashed line shows the measurement result. To calculate the transmission loss (S21) of finite periodic modulated line, equation (10) was used.

$$S_{21} = \frac{2}{2\cos(\beta d \cdot n) + j\left(\frac{Z_B}{50} + \frac{50}{Z_B}\right)\sin\left(\beta d \cdot n\right)} \quad , \tag{10}$$

where Z_B is Bloch impedance, n is the number of the period. In

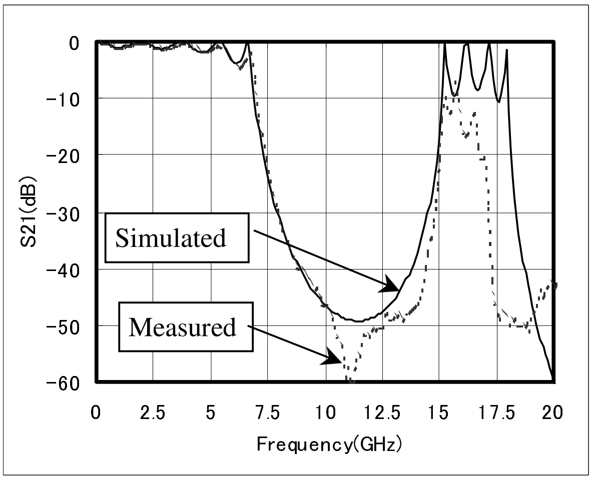


Fig.5 Simulated and experimental result of periodically loaded transmission line

fig.5, n = 5 and Z_B is a function of frequency and it was calculated by eq. (9). In the first pass band, ringing amplitude increases with frequency, which corresponds to becoming the Bloch impedance lower. In the band, simulated and measured results were well agreed each other in ringing amplitude and period. In the first stop band, simulated and measured results were well agreed each other again excepting for unwanted resonance, which was seen in measured result. The depth of the valley in stop band is proportional to the number of the period. In the second pass band, the tendency of the simulated results and measured result were well agreed each other, but amplitude of the measured result was less than simulated result. It was because the existence of coplanar wave guide leaky mode. The amplitude of the ringing was larger compared to that one in first band. It is coincide with Bloch impedance behavior, which was seen in fig.4. In this case, Bloch impedance was around 80 ohms in the second pass band, and mismatch reflection increased. It is a matter of designing to match a Bloch impedance to relevant frequency range to system impedance, which is usually 50 ohms.

V. CONCLUSION

The frequency response of the Bloch impedance was investigated by using a simple model. Experimental result validated its usefulness. Until now, phase shift and band stop response was focused on the periodic (PBG) structure. Through this work, interesting feature of the Bloch impedance over the bands was clarified. It can be used not only filter or switch but also the wide variety of (adaptive) matching circuits for the intelligent systems.

VI. ACKNOWLEDGEMENT

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VII. REFERANCES

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